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Short-term hydropower production planning by stochastic programming

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Abstract

Within the framework of multi-stage mixed-integer linear stochastic programming we develop a short-term production plan for a price-taking hydropower plant operating under uncertainty. Current production must comply with the day-ahead commitments of the previous day which makes short-term production planning a matter of spatial distribution among the reservoirs of the plant. Day-ahead market prices and reservoir inflows are, however, uncertain beyond the current operation day and water must be allocated among the reservoirs in order to strike a balance between current profits and expected future profits. A demonstration is presented with data from a Norwegian hydropower producer and the Nordic power market at Nord Pool.

Keywords: OR in energy; hydropower; stochastic programming; scenarios

1 Introduction

As is also the case in other regions of the world, hydropower production accounts for a significant share of the total power production in the Nordic countries. Indeed, the countries within Nordel produced 191 TWh hydropower out of a total production of 387 TWh in 2004 [1].

In the process of planning hydropower production, problems are usually categorized according to their time horizon, i.e. long-term, medium-term and short-term. Short-term hydropower production planning mainly involves the physical operation of the plant within a time horizon of a day or a week and with a time resolution of an hour or shorter. The most important activities that come into play are

- The day-ahead commitments, i.e. the bidding of the production into a power exchange a day in advance.

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[1]Reference: `www.nordel.org`

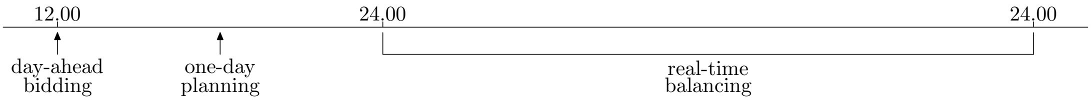

Figure 1.1: Time schedule.

- The establishment of a production plan that complies with the day-ahead commitments.
- The real-time balancing, i.e. the continuous corrections of deviations between the commitments and the actual production.

The focus of the present paper is the establishment of a one-day production plan that complies with the day-ahead commitments of the previous day. From the perspective of this paper, the day-ahead bidding has been completed whereas real-time balancing considerations will be postponed until actual production has been observed. For an illustration of the time schedule, see Fig. 1.1. When the results of the day-ahead auction are known, the classical hydropower problem, i.e. the scheduling of water *through time*, is no longer an issue. The challenge in making a production plan for the following operation day is rather the scheduling of water *through space*, i.e. the allocation of production between various parts of the plant to achieve effective and efficient operation. However, since short-term planning is strongly coupled to long-term planning the value of current decisions must be evaluated against future consequences. To determine the spatial distribution of the following operation day it is therefore common to consider production in a longer time span, e.g. seven days.
Day-ahead market prices and reservoir inflows are both subject to data uncertainty caused by non-anticipated market conditions and unpredictable weather situations. The stochastic programming framework is proposed to handle this data uncertainty. Indeed, as information evolves over time and uncertainty is disclosed in stages, a multi-stage stochastic program is appropriate. The first stage relates to the one-day production plan and the remaining stages to the production of the following six days. The overall objective of the stochastic program is to establish a one-day production plan that strikes a balance between current profits and expected future profits subject to a number of operational constraints.
Existing approaches to short-term production planning comprise both simulation and optimization. As simulation is based on adjusting manual suggestions until a convincing plan is found this approach is highly user dependent and does not guarantee an optimal plan. On the contrary, optimization represents a relatively impartial way of identifying an optimal plan. Until now however the procedures used in practice do not take data uncertainty explicitly into account.
The outline of the paper is the following. Sections 2 and 3 present a mixed-integer linear programming problem for the development of a one-day production plan that complies with the day-ahead commitments. Section 4 introduces data uncertainty and presents a stochastic programming formulation of the problem whereas Section 5 explains how to generate scenarios that serve as input. Finally, Section 6 illustrates with a case study.

We give a selected overview of related work in the field of energy optimization problems. If the inherent data uncertainty is acknowledged, a survey on the modeling of such problems by means of stochastic programming can be found in [46]. In general, the literature

covers reservoir management problems, hydrothermal coordination problems as well as hydropower production problems.
The former kinds of problems concern the management of reservoir storage levels through the scheduling of water releases in a network of reservoirs. The challenge is that releases upstream contribute to inflows downstream possibly with a time delay. The complexity of the problems increases with the stochastic nature of natural inflows. For an overview on the subject, see [27] and for applications of stochastic programming, see [37] and [47].
Hydrothermal coordination problems facilitate the combined operation of hydropower and thermal power production. Combined operation is relevant as hydropower production is relatively flexible whereas thermal is more stable. Examples of deterministic problems include [48] and [43] and stochastic problems that rest on stochastic programming are [5], [9], [15], [30] and [16].
The literature on hydropower production problems concerns the operation of a plant in order to maximize production profits earned from market disposals and potentially involves the inclusion of market price uncertainty. In continuous versions, production decisions deal with water scheduling as it is the case in [23], [45], [7] and [36]. In mixed-integer linear formulations, unit commitment decisions are included. Finally, hydropower production problems may involve bidding as in [12].
Hydropower production problems involving unit commitment have been addressed only few times in the literature. The authors of [19] and [14] present a short-term production scheduling problem of a hydropower system modeled as a deterministic mixed-integer linear program whereas [34] considers the problem subject to uncertain demand and formulates a multi-stage stochastic mixed-integer linear program.

2 Short-term hydropower production

The starting point for modeling is short-term hydropower production. Modeling is restricted to mixed-integer linear programming and follows the lines of for example [34]. For illustration purposes, the case study is kept rather simple and the model concerns only a very small hydropower plant. Some examples of nonmodeled features are constraints that apply to the network of watercourses and junctions, the distinction between baseload and loadfollowing power stations, reserve requirements as well as legal requirements, see [23]. Nevertheless, it should be clear that including additional hydrological constraints or modeling a larger hydropower plant is possible in the case of mixed-integer linear programming. The hydropower plant of the case study consists of two reservoirs in a cascade, i.e. a larger upper reservoir and a smaller lower reservoir. Each reservoir has an inflow stream. Furthermore, each reservoir is connected to a power station that contains a turbine. As upstream water reaches the plant, it is stored in the reservoirs until released through the turbines in which electricity is generated by changing the potential energy of the water into electrical energy before it proceeds downstream. Water released from the upper reservoir flows to the lower reservoir, possibly with a time delay on its way. Water that is not discharged on purpose and used for generation is considered spill. The size differences of the reservoirs restrict the flexibility of the system in that water releases from the upper reservoir may force releases in the lower reservoir or may even lead to spill. For an illustration of the plant, see Fig. 2.1. The model is presented in slightly more general terms than the case study.
The time horizon covers the current operation day for which a production plan should

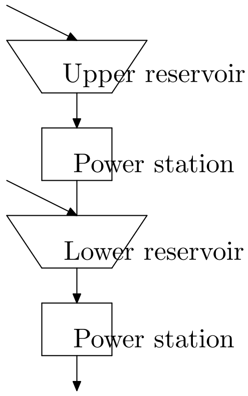

Figure 2.1: The two reservoirs in a cascade.

be made as well as the following six operation days. Due to the availability of data, the time horizon is discretized into intervals with the length of an hour and is denoted by $\mathcal{T} = \{1, \ldots, T\}$ with $T = 7 \times 24 = 168$.
To model hydropower generation, let $\mathcal{J}$ index the reservoirs and let $\mathcal{I}_j, j \in \mathcal{J}$ index the generators of the connected power stations. For the case study $\mathcal{J} = \{1, 2\}$ and $\mathcal{I}_1 = \{1\}, \mathcal{I}_2 = \{2\}$. Let the variables $u_{it} \in \{0, 1\}, i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$ represent the on/off states of the generators, $w_{it} \in \mathbb{R}_+, i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$ the generation levels and $v_{it} \in \mathbb{R}_+, i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$ the corresponding discharges from the reservoirs. Also, let the variables $l_{jt} \in \mathbb{R}_+, j \in \mathcal{J}, t \in \mathcal{T}$ be the reservoir storage levels and $r_{jt} \in \mathbb{R}_+, j \in \mathcal{J}, t \in \mathcal{T}$ the spill.
As concerns direct costs of hydropower generation, operating costs are negligible. However, start-up costs amount to

$$\sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_j} S_i(u_{it-1}, u_{it})$$

where the cost functions are

$$S_i(u_{it-1}, u_{it}) = c_i \max\{u_{it} - u_{it-1}, 0\}, \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$$

and the costs per start-up are $c_i, i \in \mathcal{I}_j, j \in \mathcal{J}$. It should be remarked that the formulation corresponds to a mixed-integer linear formulation. Initial on/off states of the generators are $u_{i0} = u_{i,init}, i \in \mathcal{I}_j, j \in \mathcal{J}$.
Indirect costs include opportunity costs of releasing water as the water could be stored and saved for future generation and thus such costs are measured as the value of stored water. The opportunity costs are

$$\sum_{j \in \mathcal{J}} (V_j(l_{j0}) - V_j(l_{jT}))$$

where

$$V_j(l_{jt}) = \min_{h \in \mathcal{H}} \{e^1_{hj} l_{jt} + e^2_{hj}\}, \;\; j \in \mathcal{J}, t \in \mathcal{T}$$

and the coefficients of the concave water value functions are $e^1_{hj}, e^2_{hj}, h \in \mathcal{H}, j \in \mathcal{J}$. The formulation is consistent with a linear formulation.
As concerns the case study, the water value of the upper reservoir accounts for the opportunity of releasing water from both the upper and the lower reservoirs. However, the

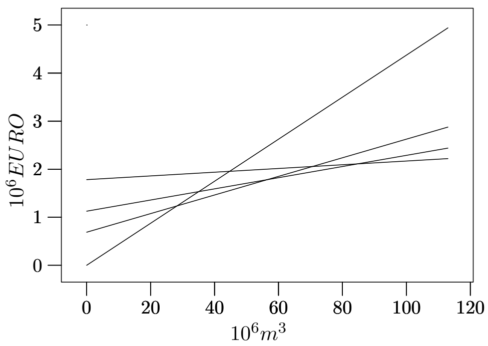

Figure 2.2: Water value function of the upper reservoir.

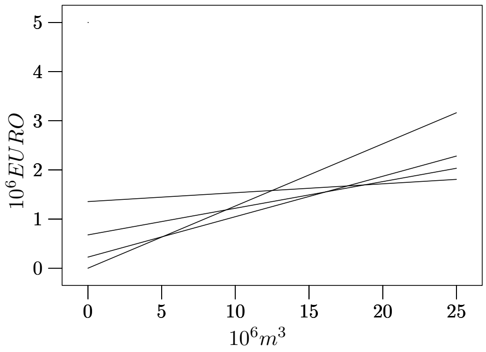

Figure 2.3: Water value function of the lower reservoir.

possibility of releasing water from the upper reservoir depends on the level of the lower reservoir. With large size differences, possibilities of releasing water from the upper reservoir might be limited and the water value of the upper reservoir might be even lower than that of the lower reservoir. This is the case in Figs. 2.2 and 2.3. The same principle applies with more than two reservoirs.
The following bounds are imposed on the generation levels

$$w_i^{min} u_{it} \leq w_{it} \leq w_i^{max} u_{it}, \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T} \tag{2.1}$$

in which $w_i^{min}, i \in \mathcal{I}_j, j \in \mathcal{J}$ and $w_i^{max}, i \in \mathcal{I}_j, j \in \mathcal{J}$ are minimum and maximum generation levels. The water discharges have to comply with similar bounds, i.e.

$$v_i^{min} \leq v_{it} \leq v_i^{max}, \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T} \tag{2.2}$$

in which $v_i^{min}, i \in \mathcal{I}_j, j \in \mathcal{J}$ and $v_i^{max}, i \in \mathcal{I}_j, j \in \mathcal{J}$ are the minimum and maximum discharges. Finally, the following bounds apply to the storage levels

$$l_j^{min} \leq l_{jt} \leq l_j^{max}, \;\; j \in \mathcal{J}, t \in \mathcal{T} \tag{2.3}$$

where $l_j^{min}, j \in \mathcal{J}$ and $l_j^{max}, j \in \mathcal{J}$ denote minimal and maximal storage levels.
The power generated is a function of the water discharge from the reservoir and the net water head of the power station. Whereas the headwater elevation is a function of the reservoir storage level, the tailwater elevation is a function of the discharge. It is however assumed that the net water head does not vary much with the reservoir storage level over the course of the short-term planning horizon. The assumption is justified in the case of relatively small storage level variations, which holds for the case study. Ignoring head variation effects, the relation between generation and discharge is approximated fairly well by a concave function. Hence,

$$w_{it} = G_i(v_{it}), \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T} \tag{2.4}$$

where

$$G_i(v_{it}) = \min_{h \in \mathcal{H}} \{ f_{hi}^1 v_{it} + f_{hi}^2 \}, \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, t \in \mathcal{T}$$

and the coefficients of the concave functions are $f_{hi}^1, f_{hi}^2, h \in \mathcal{H}, i \in \mathcal{I}_j, j \in \mathcal{J}$.
According to the reservoir balances, inflow and storage from previous periods either appear as discharge, storage or spill in the following period. The upper reservoir balances are

$$l_{1t} - l_{1t-1} + \sum_{i \in \mathcal{I}_1} v_{it} + r_{1t} = \nu_{1t}, \;\; t \in \mathcal{T} \tag{2.5}$$

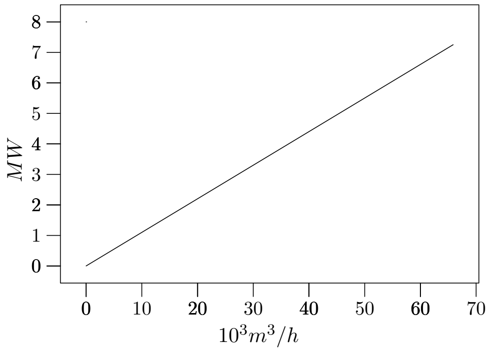

Figure 2.4: Power generation function of the upper reservoir.

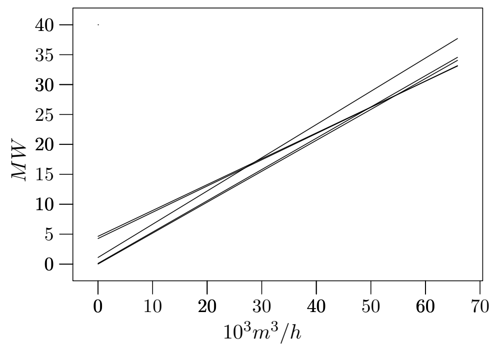

Figure 2.5: Power generation function of the lower reservoir.

in which $\nu_{1t}, t \in \mathcal{T}$ are the inflows from upstream. The initial storage level is $l_{10} = l_{1,init}$. The lower reservoir balances are

$$l_{jt} - l_{jt-1} + \sum_{i \in \mathcal{I}_j} v_{it} + r_{jt} = v_{j-1t-\tau} + \nu_{jt}, \;\; j \in \mathcal{J}\backslash\{1\}, t \in \mathcal{T} \tag{2.6}$$

in which $\nu_{jt}, j \in \mathcal{J}\backslash\{1\}, t \in \mathcal{T}$ are the direct inflows from upstream. Note that releases from the upper reservoirs are inflows of the lower reservoirs. τ is the time delay between the reservoirs. Again, the initial storage level is $l_{j0} = l_{j,init}, j \in \mathcal{J}\backslash\{1\}$.

3 Day-ahead market commitments

Although the case study concerns a Norwegian hydropower producer, the model applies to any price-taking day-ahead market participant. As regards the deregulation of the power markets, Norway was among the first countries in the world to undertake the process. In the beginning of the nineties a Norwegian power market was established and has since that time developed into an overall Nordic power market. An essential component of the power market is the presence of the power exchange facilitating physical trading activity on a day-per-day basis. The spot market, Elspot, at Nordic power exchange, Nord Pool, is a pool-based market in which market participants exchange power contracts for physical delivery the following operation day and is referred to as the day-ahead market. In 2004 a total of 167 TWh of the Nordel power production was traded at Elspot which represents 43 percent [2].

Elspot contracts are commitments to sell or purchase power of a duration of one hour or longer. The market participants post price-differentiated bids for every hour of the following operation day before deadline at noon. The hourly market prices are determined by equilibrium between sales and purchases. Once the market prices have been announced, the market participants receive a notification of the winning bids and the hourly commitments of the following operation day. Real-time operation and physical delivery is done according to the day-ahead commitments to the extent possible.

The value of electricity production should be measured on the basis of day-ahead market prices. Without loss of generality, it is assumed that the entire production is sold in the day-ahead market and that more long-term bilateral contracts are left out. Day-ahead

[2] Reference: `www.nordel.org` and `www.nordpool.no`

market sales give rise to the revenues

$$\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_j}\rho_t w_{it}$$

where $\rho_t, t \in \mathcal{T}$ are the day-ahead market prices. By assuming that the producer is a price-taker, market prices can be modeled as exogenous. To justify the price-taker assumption, the producer is assumed to be sufficiently small to be of limited market power. In the case of considerable market power the concepts of game theory, monopoly or oligopoly becomes important and the complexity of the model may increase.
The production of the first day has to meet the hourly commitments in the day-ahead market fixed the day before. As the day-ahead commitments are fixed the day before, these are given as data of the model. This gives rise to the constraints

$$\sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_j} w_{it} = d_t, \;\; t \in \mathcal{T}_1 \tag{3.1}$$

where $\mathcal{T}_1$ index the hours of the first day and $d_t, t \in \mathcal{T}_1$ are the day-ahead commitments of the first day. Note that in the case of portfolio bidding, i.e. if submitting compound bids for all plants of a producer, the modeling of a larger hydropower system comes into play. Still, it is straightforward to extend the model.
If the production deviates from the day-ahead commitments, the imbalances are settled in a real-time balancing market. Since the purpose of the model is production planning rather than market exchange, however, real-time balancing is not included. Moreover, since planned imbalances are not allowed by the market operators, producers should not have incentives to hold back production for the balancing market. Nevertheless, although real-time balancing is left out from the process of production planning, production can always ramp down or, in the case of spare capacity, ramp up while actually producing and participate in the real-time balancing market.

4 The stochastic programming model

Both day-ahead prices and water inflows are rather volatile and hard to predict because of unexpected market conditions and unforeseen weather situations. The model of the previous sections does not reflect the fact that new information on the uncertain data arrives as time evolves along the planning horizon. Nevertheless, this can be handled by the application of multi-stage stochastic programming.
Basically, stochastic programming deals with optimal decision making under uncertainty over time. In a multi-stage setting the outcome of uncertain data is only gradually revealed and decisions are made dynamically without anticipating the future outcome. For an introduction to stochastic programming in general and multi-stage stochastic programming specifically, see [2], [24] and [35]. Provided that the relevant probability information is available, the current model can be formulated as a multi-stage stochastic program.
The uncertain data evolves over time according to a multivariate stochastic process and the probability information is approximated by a so-called scenario tree. The root node corresponds to time interval $t = 1$. The remaining nodes all have an ascendant node and a set of descendant nodes. For node n the immediate ascending node is termed n_{-1} with the transition probability $\pi^{n/n_{-1}}$, i.e. the probability that n is the descendant of n_{-1}. The probabilities of the nodes are given recursively by $\pi^1 = 1$ and $\pi^n = \pi^{n/n_{-1}}\pi^{n_{-1}}, n > 1$.

The ascendant node of node n at t time intervals back in time is n_{-t}. The immediate descendants of node n are $\mathcal{N}_{+1}(n)$ and nodes with $\mathcal{N}_{+1}(n) = \emptyset$ are called leaves. Moreover, the path from the root node to node n is denoted by $path(n)$ and $t(n)$ is its length. $\mathcal{N}_t$ is the set $\{n \in \mathcal{N} : t(n) = t\}$ and nodes of $\mathcal{N}_T$ constitute the leaves. Each path from the root node to a leaf represents a scenario. For $t \in \mathcal{T}$ the realizations of the uncertain data $\{\nu_{1t}, \nu_{2t}, \rho_t\}$ are denoted $\{\nu_1^n, \nu_2^n, \rho^n\}_{n \in \mathcal{N}_t}$.
By assuming that information is revealed only at the beginning of an operation day, the scenario tree consists of seven stages or operation days that each consists of 24 time intervals or hours. The assumption is valid for day-ahead market prices that are announced a day prior to physical delivery. Furthermore, it is justified in the case of daily measurements of water inflows or at least as an effort to limit the size of the scenario tree. An example of a scenario tree is shown in Fig. 4.
The inclusion of seven stages makes for the coupling between short-term and long-term

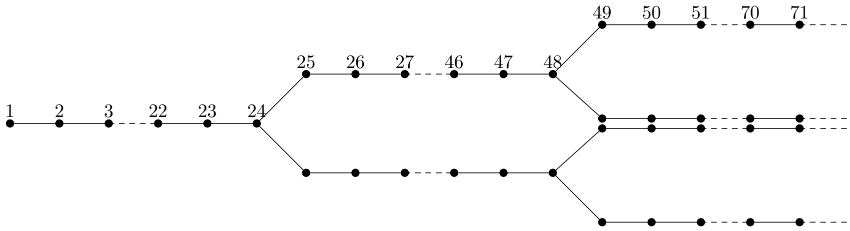

Figure 4.1: Part of a scenario tree.

planning. While the first stage determines the one-day production plan, the remaining six stages serve to evaluate the impact of the one-day production plan on future production. Indeed, the overall objective of the multi-stage stochastic program is to determine the one-day production plan that strikes a balance between current profits and expected future profits. The multi-stage stochastic program is the following

$$\max \sum_{n \in \mathcal{N}} \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_j} \pi^n \rho^n w_i^n - \sum_{n \in \mathcal{N}} \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_j} \pi^n S_i(u_i^{n-1}, u_i^n) + \sum_{j \in \mathcal{J}} V_j(l_{j0}) - \sum_{n \in \mathcal{N}_T} \sum_{j \in \mathcal{J}} \pi^n V_j(l_j^n) \tag{4.1}$$

$$\sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_j} w_i^n = d^n, \;\; n \in \mathcal{N}_t, t \in \mathcal{T}_1$$

$$u_i^n w_i^{min} \leq w_i^n \leq u_i^n w_i^{max}, \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, n \in \mathcal{N}$$

$$v_i^{min} \leq v_i^n \leq v_i^{max}, \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, n \in \mathcal{N}$$

$$l_j^{min} \leq l_j^n \leq l_j^{max}, \;\; j \in \mathcal{J}, n \in \mathcal{N}$$

$$l_1^n - l_1^{n-1} + \sum_{i \in \mathcal{I}_1} v_i^n + r_1^n = \nu_1^n, \;\; n \in \mathcal{N}$$

$$l_j^n - l_j^{n-1} + \sum_{i \in \mathcal{I}_j} v_i^n + r_j^n = v_1^{n-\tau} + \nu_j^n, \;\; j \in \mathcal{J} \backslash \{1\}, n \in \mathcal{N}$$

$$w_i^n = G_i(v_i^n), \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, n \in \mathcal{N}$$

$$u_i^n \in \{0,1\}, w_i^n, v_i^n, l_j^n, r_j^n \geq 0, \;\; i \in \mathcal{I}_j, j \in \mathcal{J}, n \in \mathcal{N}$$

Note that the multi-stage stochastic program may be approximated by a two-stage program. The natural way to construct the two-stage approximation would be to let the first stage determine the one-day production plan and the second stage consist of the remaining six days.

5 Scenario generation

The modelling of day-ahead market prices and water inflows is based on time series analysis. Time series models can be derived from limited data and information and still allow for forecasting and simulating. In practice, power producers use socalled fundamental models that specify detailed physical relationships. Although the forecast performance of fundamental models may be superior, these are not constructed for simulating which is needed for scenario generation. The use of ARMA models and variants of these to forecast hourly day-ahead market prices are often seen in the litterature, see [8], [29], [13] and [17] for examples. ARMA models however have been used mostly to forecast monthly and annual water inflows, whereas hourly inflows modeled by the ARMA framework has occured only rarely in the litterature. Alternatives to the ARMA framework may be better suited to take into account the effect of sudden jumps due to heavy rainfalls, persistence caused by the tendency for weather conditions to continue for some period of time and many high level crossings in which inflows remain above or below a given threshold. Nevertheless, ARMA models are considered to be sufficient for the scenario generation in order to demonstrate the usefulness of stochastic programming.

The multivariate stochastic process of hourly day-ahead market prices and water inflows constitute a time series characterized by seasonal changes, periodic cycles and stochastic variations [3]. Day-ahead market prices are mainly driven by supply and demand patterns whereas the most significant features of water inflows are attributed to precipitation behavior. Box and Jenkins [3] introduced the ARMA processes, which form a class of stochastic processes suitable for describing such time series. In general, a multivariate stochastic process can be modeled as a vector ARMA process

$$\Phi(B)X_t = \Theta(B)\mathrm{E}_t, \;\; t \in \mathbb{Z}$$

where $\Phi(B)$ and $\Theta(B)$ are the polynomials $\Phi(B) = 1 - \sum_i \phi_i B^i$ and $\Theta(B) = 1 - \sum_i \theta_i B^i$ with the parameter matrices ϕ_i and θ_i and where B denotes the back-shift operator, i.e. $B^i X_t = X_{t-i}$. X_t and E_t are vectors where $\mathrm{E}_t, t \in \mathbb{Z}$ are assumed to be independent identically and normally distributed random vectors with the covariance matrix Σ.

Consider the three-dimensional stochastic process $\{\nu_{1t}, \nu_{2t}, \rho_t\}_{t\in\mathcal{T}}$ of day-ahead market prices and water inflows. An inspection of the cross correlations indicates that the price series and the inflow series are uncorrelated and therefore can be modeled separately and that the inflow series are only contemporaneously correlated. If contemporaneously correlated, any interaction between the inflows series is instantaneous. The approach is a standard way of modeling multivariate stream-flow series. It is motivated by the fact that vector ARMA models of higher orders are difficult to estimate and that a contemporaneous model allows model decoupling into univariate components. For further references on contemporaneous ARMA models and stream-flows, see [1], [4], [39] and [40]. As a result three univariate ARMA models may be fitted independently and combined to a multivariate model.

The development of the univariate ARMA models follows the steps

1. Identify a statistical model of the data observed.

[3] Data sources: Nord Pool (prices) and TrønderEnergi (inflows)

2. Estimate the parameters of the model.

3. Validate the model.

1. In the identification step the data is made stationary by differentiation and the orders of the polynomials are determined by investigating correlations. The ARMA models are identified as

$$(1-\phi_1 B)(1-B)(1-B^{24})(1-B^{168})\rho_t = (1-\theta_1 B)(1-\theta_{24}B^{24})(1-\theta_{168}B^{168})\epsilon_t, \ \ t \in \mathbb{Z} \quad (5.1)$$

for day-ahead market prices and

$$(1-\phi_1^j B)(1-B)\nu_{jt} = (1-\theta_1^j B - \theta_2^j B^2)(1-\theta_{41}^j B^{41})\epsilon_{jt}, \ \ j=1,2, t \in \mathbb{Z} \quad (5.2)$$

for water inflows.
2. Parameter estimation is based on maximum likelihood optimization. The estimates can be found in Tables 1 and 2.
3. Finally, diagnostic checks are applied to the residuals to validate the assumptions of independence and normality.
The residuals of (5.1) and (5.2), i.e. $\{\epsilon_t\}_{t\in\mathcal{T}}$ and $\{\epsilon_{1t}, \epsilon_{2t}\}_{t\in\mathcal{T}}$, are uncorrelated and, since normally distributed, independent. The residuals of (5.2), i.e. $\{\epsilon_{1t}\}_{t\in\mathcal{T}}$ and $\{\epsilon_{2t}\}_{t\in\mathcal{T}}$, are contemporaneously correlated. In order to obtain completely uncorrelated residuals, the transformation $(\epsilon_{1t}, \epsilon_{2t})^T = C(\bar{\epsilon}_{1t}, \bar{\epsilon}_{2t})^T$ is performed where $CC^T = \Sigma$, C is an upper triangular matrix, Σ is the covariance matrix of the residuals and $\{\bar{\epsilon}_{1t}, \bar{\epsilon}_{2t}\}_{t\in\mathcal{T}}$ are independent identically and normally distributed random vectors. The entries of the estimated covariance matrix are listed in Table 3. A multivariate ARMA model that describes the three-dimensional process of day-ahead market prices and water inflows has been fitted. The fitting of the time series models, i.e. steps 1-3, is accomplished by the statistical software package SAS, version 8.2.

Table 1: Parameter estimates of univariate ARMA model, hourly day-ahead market prices

Par.	ϕ_1	θ_1	θ_{24}	θ_{168}	Σ
Est.	0.6874	0.9234	0.8502	0.9665	5.9386

Table 2: Parameter estimates of univariate ARMA models, hourly water inflows

Par.	ϕ_1^1	θ_1^1	θ_2^1	θ_{41}^1	ϕ_1^2	θ_1^2	θ_2^2	θ_{41}^2
Est.	0.9899	1.3156	−0.3504	0.8424	0.9775	1.4442	−0.5509	0.8304

Table 3: Parameter estimates of the multivariate ARMA model, hourly water inflows

Par.	Σ_{11}	Σ_{12}	Σ_{21}	Σ_{22}
Est.	8577924	90712	90712	542625

There are various approaches to approximate what is actually a continuous probability distribution of the uncertain data by a discrete distribution represented by a scenario tree. For a general survey on constructing scenario trees, see for instance [11]. Examples cover

internal sampling methods, [22], moment matching principles, [20] and [21], Monte Carlo based approaches, [42], Quasi Monte Carlo based approaches, [32] as well as methods motivated by stability analysis, [33] and [18]. The starting point of the present approach is to generate 1000 scenario paths that are Monte Carlo samples from the fitted multivariate ARMA model. Some descriptive statistics of the generated scenarios are given in Table 4 for a sample size of 10000. Examples of a few scenario paths are shown in Figs. 5.1, 5.2 and 5.3. The individual scenario paths are combined to a scenario tree by applying the method of [18]. The scenario tree construction is conducted by a heuristic that works recursively either backwards or forwards stage by stage bundling the scenario paths at the current stage. For multi-stage stochastic linear programs, the method is supported by stability results, i.e. the optimal value depends continuously on the stochastic data. Consequently, the quality of the constructed scenario tree can be controlled by certain error bounds. The method has been implemented in a test version most kindly made available by the authors. In approximating a continuous probability distribution by a discrete distribution, the quality of the approximation is directly linked to the quality of the scenario generation method. As the error bounds of the constructed scenario tree are often quite loose, the performance of the scenario generation method may also be evaluated in terms of in-sample stability as well as out-of-sample stability and tested for the introduction of potential bias into the solution, cf. [26]. Stability has been tested and found to be satisfactorily fulfilled.
In addition to simulation scenarios, demonstration scenarios have been generated to illustrate various effects of day-ahead prices and water inflows. In the demonstration scenarios, prices and inflows are allowed to be significantly higher and lower than real observations of the data.

Table 4: Descriptive statistics of hourly day-ahead market prices and water inflows

		Mean	Std. dev.	Min.	Max.
Prices	NOK/MWh	174.27	26.14	44.08	298.95
Inflow, upper res.	m^3/h	24591.62	22198.22	0.00	162381.10
Inflow, lower res.	m^3/h	10788.66	10217.04	0.00	72739.46

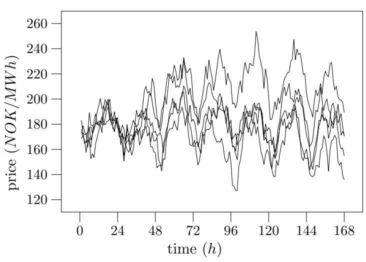

Figure 5.1: Hourly day-ahead market price scenarios.

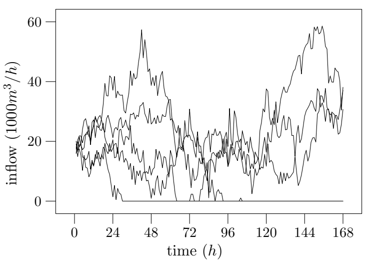

Figure 5.2: Hourly water inflow scenarios, upper reservoir.

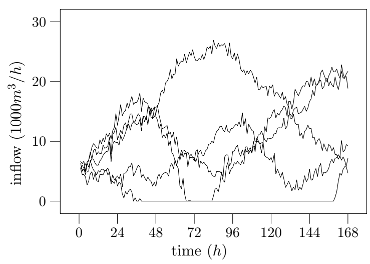

Figure 5.3: Hourly water inflow scenarios, lower reservoir.

6 Computational results

The case study is based on data from a small Norwegian hydropower plant located south of Trondheim and run by the company TrønderEnergi. The annual inflows to all power

plants of this company amount to 1.5 TWh, the generation capacity to 334 MW and the reservoir capacity to 0.7 TWh [4]. However, the case study concerns only two reservoirs which corresponds to annual inflows of 69.6 GWh, a generation capacity of 33.9 MW and a reservoir capacity of 24.6 GWh. The initial conditions of the plant are given by data from a typical day of 2005. The same applies for the day-ahead commiments. The generation of water inflows for 2005 is based on observations from the year 2004. To generate corresponding day-ahead market prices for 2005, Nord Pool has provided data from 2004 that applied to the Norwegian prize zone, NO2, which includes the Trondheim area. Basically, NO2 consists of nine hydropower producers aside from a number of very small producers. The company TrønderEnergi accounts for 5 percent of the annual inflow, 5 percent of the generation capacity and 3 percent of the reservoir capacity of the area [5] and is therefore considered a price-taker of the area.
The multi-stage stochastic program (4.1) contains 12 variables, 1 binary and 11 continuous, and 22 constraints per node of the scenario tree except for a few extra variables and constraints in the first and the last stages. Hence, it is a mixed-integer linear program whose size depends on the number of scenarios and nodes of the scenario tree. We have solved the problem with the mixed-integer linear programming solver from OPL Studio version 3.7 calling CPLEX 9.0 on an Intel Xeon 2.67 GHz processor with 4 GB RAM. Direct application of CPLEX is not possible when the number of scenarios and nodes of the scenario tree is further increased. However, the problem is suitable for decomposition approaches that are often based on Lagrangian relaxation of nonanticipativity constraints, cf. [6], [28] and [44], nodal coupling constraints, cf. [10], or component coupling constraints, cf. [9], [15], [16] and [30]. While varying the level of bundling in the scenario tree construction method we have recorded the number of scenarios and nodes of the scenario tree, the total number of variables and constraints of the problem, the optimal objective function value and the computing time spent to solve the problem, cf. Table 5. All numbers reported are averages of 10 different runs and all computations are based on simulation scenarios. It should be remarked that test runs show that the two-stage approximation to the multi-stage stochatic program may provide very good first-stage solutions in terms of objective function values.
As regards hydropower production, planning is coupled in time. The time coupling can

Table 5: Computational results, simulation scenarios

Sce.	Nodes	Var.	Con.	Obj. val./NOK	CPU/s
267	11777	141846	261254	562207.50	28.71
493	43709	525494	965566	567086.91	203.81
782	103144	1239304	2275470	566645.83	764.35

mainly be explained by to the storage balancing between the reservoirs. The storage balancing and the capacities of the reservoirs determine the spatial distribution of water between the reservoirs of the hydropower plant. The spacial distribution therefore depends on future day-ahead market prices and water inflows to the reservoirs. To see this, consider the case where both reservoirs are close to empty. If day-ahead market prices for the following six days are expected to be high, there is a potential for future generation and current production takes place in downstream reservoirs. Indeed, this ensures future

[4] Norwegian Competition Authority, 2002
[5] Norwegian Competition Authority, 2002

water releases from all reservoirs in the cascade and higher future generation levels. If day-ahead market prices for the following six days are expected to be low, current production is allocated according to water values and start-up costs. Moreover, consider the case where both reservoirs are almost full. If water inflows for the following six days are expected to be high, current production takes place in downstream reservoirs in order to empty the system and accommodate future generation. If water inflows for the following six days are expected to be low, current production is allocated according to water values and start-up costs.

The dependence of the spatial distribution on the future day-ahead market price and water inflow level is illustrated in Figs. 6.1-6.4. The columns of the figures represent the hourly day-ahead commitments which are chosen to be constant and the dark and light gray colours demonstrate the distribution of generation between the upper and lower reservoirs. The illustrations have been based on demonstration scenarios.

Obviously, mixed effects of future day-ahead market prices and water inflows may appear. If both reservoirs are nearly empty and low day-ahead market prices are expected, current production may take place upstream or downstream depending on future inflow levels since current production is allocated according to water values and start-up costs. In the same fashion, consider the case where both reservoirs are close to full and water inflows are expected to be low. If low day-ahead market prices are expected, current production takes place upstream. If day-ahead market prices are expected to be high, there is a potential for future generation and current production takes place both upstream and downstream to ensure higher future generation levels.

Time coupling may also be explained by start-up costs. Besides, start-up costs contribute

Spatial distribution among reservoirs, both reservoirs are close to empty.

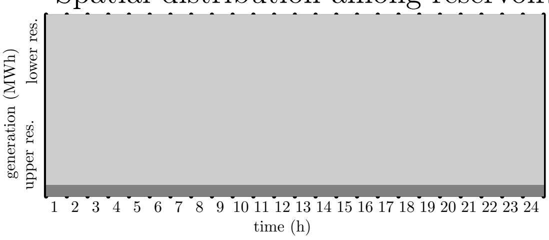

Figure 6.1: High day-ahead price level.

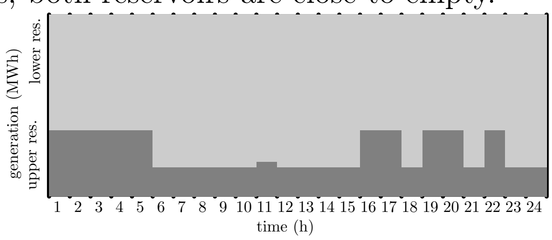

Figure 6.2: Low day-ahead price level.

Spatial distribution among reservoirs, both reservoirs are close to full.

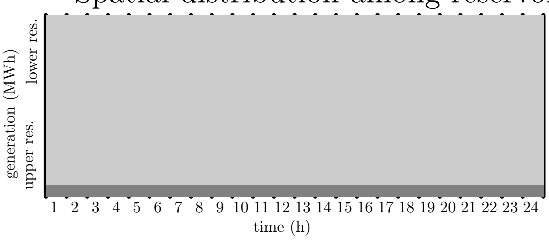

Figure 6.3: High water inflow level.

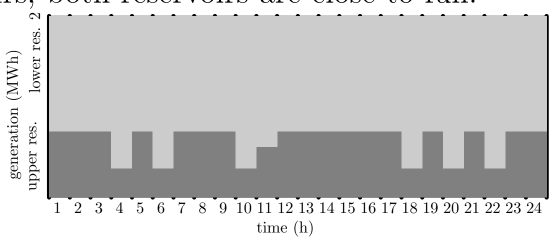

Figure 6.4: Low water inflow low.

to the complexity of the model in that binary variables are introduced. By the use of simulation scenarios the result of the case study is a production plan with few start-ups. Therefore, to illustrate the effect of start-up costs, we have generated demonstration scenarios that capture fluctuations in day-ahead prices and water inflows and used these for testing. The results of fluctuating day-ahead market prices alone and both fluctuating day-ahead market prices and water inflows are displayed in Table 6. We have listed the optimal objective function value, the expected number of start-ups and the expected start-up costs. Although start-up costs seem to be limited, the mixed-integer linear formulation still has relevance since modeling a larger hydropower plant with more turbines or including other hydrological constraints may involve integers. Moreover, increasing the cost per start-up, total start-up costs increase even though the expected number of start-ups decreases.

By employing the expectation-based objective function criterion, the production planning

Table 6: Start-up costs

	Obj. val./NOK	Start-ups	Start-up costs/NOK
Prices	1466433.58	0.34	390.53
Prices and inflows	1462328.61	0.22	214.62

is conducted in a risk neutral fashion. As most power producers are in fact risk averse, portfolio hedging comes into play. Portfolio hedging is often separated from production planning so that the aim of planning is to maximize the value of the available resources while hedging alone aims to control the risk of the portfolio. Still, to control risk along with production planning, and in particular achieve a more uniform profit distribution among scenarios, a risk measure can be appended to the objective function. The result is a so-called mean-risk model. The downside risk measure semideviation penalizes deviations below expected profit and has the advantage of being consistent with a linear formulation. We append the risk measure and penalize accumulated deviations to obtain the following mean-risk model

$$\sum_{n\in\mathcal{N}} \pi^n z^n - \rho \sum_{n\in\mathcal{N}_T} \pi^n \max\{\sum_{\bar{n}\in\mathcal{N}} \pi^{\bar{n}} z^{\bar{n}} - \sum_{\bar{n}\in path(n)} z^{\bar{n}}, 0\}$$

where ρ is a weight and where

$$z^1 = \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_j} \rho^1 w_i^1 - \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_j} S_i(u_i^{1-1}, u_i^1) + \sum_{j\in\mathcal{J}} V_j(l_{j0})$$

$$z^n = \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_j} \rho^n w_i^n - \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_j} S_i(u_i^{n-1}, u_i^n), \;\; n \notin \mathcal{N}_T \cap \{1\}$$

$$z^n = \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_j} \rho^n w_i^n - \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_j} S_i(u_i^{n-1}, u_i^n) - \sum_{j\in\mathcal{J}} V_j(l_j^n), \;\; n \in \mathcal{N}_T$$

It should be remarked that it is possible to include a multi-period risk measure instead which do not focus on accumulated profit alone but also take into account intermediate time periods.

We have solved the problem for varying weights and recorded the expected value along with the risk, cf. Table 7. All numbers reported are averages of 10 different runs and all computations are based on simulation scenarios. As can be seen in Table 7, if employing the expectation-based objective function criterion in short-term production planning, the

Table 7: Computational results for semideviation, simulation scenarios

ρ	0.001	1	10	100	1000
Exp. val.	563110.56	563110.54	495984.66	467811.71	457764.42
Risk	21906.52	21906.26	1480.32	43.03	0.00

downside risk is significant. Also, a reduction in risk requires a considerable reduction in profit. In the long run, however, hydropower producers have a natural hedge. Whereas prices and inflow are uncorrelated in the short run they are usually negatively correlated in a longer time span. If inflows decrease, prices tend to increase and compensate for this and vice versa.

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